Improved Guidance Law Design Based on the Mixed-Strategy Concept

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The paper summarizes a new approach for missile guidance law design in a noise-corrupted environment. This approach is based on the concept of mixed strategies known from game theory. Two examples demonstrate that this innovative design approach leads to an improved missile performance compared to all guidance laws previously discussed in the open literature.

Introduction

THE terminal phase of an encounter between a radar-guided-missile and an aircraft can be formulated as a differential game with imperfect, noise-corrupted information. It has been shown that if it were possible for the missile to obtain perfect measurements of the line-of-sight (LOS) rotation rate then, in most practical situations, the miss distances would be small enough for destroying the target no matter how it would maneuver.[1,2] Fortunately for the aircraft this is not the case, because the measurements taken by a radar-guided missile are always noisy. In radar-guided missiles, the designer has to incorporate an estimator (or filter) in the guidance loop. In the design of the estimator, the designer is forced to make assumptions on the expected target maneuvering behavior. The better these assumptions match the actual maneuver, the better the overall performance of the missile. The logical conclusion from these last statements is that the optimal strategy of the target is to maneuver in an unpredictable way, which is necessarily random. In other words, the optimal strategy of the target against an optimally designed missile is mixed (a mixed strategy is a probability distribution on a pure strategy set[3]). This conclusion has been reached in the past.[4-6]

From a theoretical differential game point of view, it can be expected that if the evader's optimal strategy is mixed, then in general the pursuer's strategy also will be mixed. Nevertheless, until recently no effort had been made in the open literature in order to find optimal mixed strategies for the pursuer.

Recently, the authors of this paper introduced a new differential game formulation of the problem[7] and proposed a mathematical framework[8] for the computation of mixed optimal missile guidance strategies in a scenario with partial and noise-corrupted information.

The present paper summarizes this new approach and presents two interesting applications. The first example deals with a missile-aircraft encounter in an electronic countermeasure (ECM) free environment, whereas in the second example the aircraft makes use of ECM in the form of electronic jinking.[9]

Formulation of the Problem

In this section, the main features of the formulation detailed in Ref. 7 are summarized.

For the sake of simplicity, a two-dimensional version of the terminal phase of a missile-aircraft encounter is adopted. It is assumed that the game takes place in the vicinity of the collision course (see Fig. 1). In this section, the missile and the aircraft will be called pursuer and evader, respectively.

Throughout the duration of the game the pursuer measures the relative range R, the closing velocity V_c, and the LOS angle λ, relative to a reference line. It is assumed that the range and velocity measurements are exact and that the angle measurement is corrupted by noise. The velocity and range information is processed to give an accurate estimate of the time-to-go t_{go}, while the range and angle information is processed and yields a "noisy measurement" Z of the evader's relative position perpendicular to the reference line. The input to the estimation process is Z and it is given by:

$$Z = R\sin(\lambda + \text{noise}) = y + (w + \zeta) \tag{1}$$

where y is the evader's true relative position perpendicular to the reference line, w an intentional disturbance caused by the evader's ECM, and ζ a zero mean Gaussian measurement noise (glint).

The evader has no measurements on the state of the game and has no knowledge of the duration of the game, even though he knows when the game starts.

The pure strategy set of the pursuer Δ_p is defined as a countable set of "guidance policies" δ_{p_j} of a given structure. The pure strategy set of the evader Δ_e is defined as a (possibly infinite) set of "actions" δ_{e_i}, each of which is composed of a maneuver sequence and of some countermeasure policy (e.g., electronic jinking[9]).

The game is played as follows: At the beginning of the game, or shortly before it, each player "chooses," through a chance mechanism, one of its pure strategies and plays according to it until the end of the game. The chance mechanism, which determines the pure strategy to be played, is a mechanization of the player's mixed strategy.

It should be mentioned that, because of the short duration of the game, the selection of one of the available pure strategies at the outset seems to be the only reasonable way to play in a future operational scenario in view of the following facts:

1) It is practically impossible for the evader to determine with reasonable accuracy in real time the guidance policy used

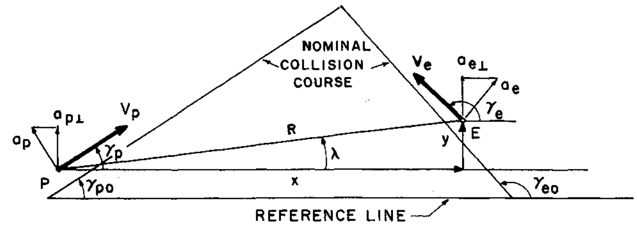

Fig. 1 Relative geometry of the game.

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by the pursuer in a particular encounter, even when taking measurements of the pursuer's flight path. Thus, once the evader has "chosen" his strategy there is no reason to change it throughout the game.

2) It is well known that the estimation of a random and unpredictable evasive motion based on noisy measurements of the displacements only is a very difficult task. It is even more difficult to make an estimate of the model or parameters which describe the actual maneuver. Thus, as long as the evader maneuvers in an unpredictable way, it is clear that the pursuer cannot take advantage of its measurements in order to verify in real time the assumption on which its estimator is based. This means that, throughout the duration of the game, the pursuer has no "basis" to change its guidance policy.

The evader's and pursuer's mixed strategies are determined by sequences of real numbers $\{\alpha_i\}$ $i=1,...,m_e$ and $\{\beta_j\}$ $j=1,...,m_p$ respectively, which satisfy

$$\sum_{i=1}^{m_e} \alpha_i = 1, \qquad \alpha_i \geq 0 \ \forall \qquad i=1,...,m_e \tag{2a}$$

$$\sum_{i=1}^{m_p} \beta_j = 1, \qquad \beta_j \geq 0 \ \forall \qquad j=1,...,m_p \tag{2b}$$

where α_i determines the probability of "choosing" δ_{e_i} by the evader and where β_j determines the probability of "choosing" δ_{p_j} by the pursuer.

The payoff function J is given in terms of Δ_e, Δ_p, $\{\alpha_i\}$, and $\{\beta_j\}$ by:

$$J = J(\Delta_e,\{\alpha_i\},\Delta_p,\{\beta_j\})$$

$$= \sum_{i=1}^{m_e}\sum_{j=1}^{m_p} \alpha_i\beta_j P_{ij} \triangleq J_{\Delta_e,\Delta_p}(\{\alpha_i\},\{\beta_j\}) \tag{3}$$

where P_{ij} is the single shot kill probability (SSKP) for the case in which the pure strategies δ_{e_i} and δ_{p_j} are played and is expressed by:

$$P_{ij} = E\{P[x_1(t_f)]|\delta_{e_i},\delta_{p_j}\} \tag{4}$$

where $x_1(t_f)$ is the miss distance and $P(\cdot)$ is a real-valued function that describes the warhead lethality subject to $0 \leq P(\cdot) \leq 1$.

The pursuer wishes to maximize J while the evader wishes to minimize it.

Given the pure strategy sets Δ_e and Δ_p, the solution of the game is presented by the optimal sequences $\{\alpha_i^*\}$, $\{\beta_j^*\}$, called the optimal mixed strategies of the evader and the pursuer respectively, and by a real number $0 \leq V_m \leq 1$ that is called the value of the game. V_m is defined by:

$$V_m \triangleq J_{\Delta_e,\Delta_p}(\{\alpha_i^*\},\{\beta_j^*\}) \tag{5}$$

The value satisfies a saddle point inequality

$$J_{\Delta_e,\Delta_p}(\{\alpha_i^*\},\{\beta_j\}) \leq V_m \leq J_{\Delta_e,\Delta_p}(\{\alpha_i\},\{\beta_j^*\}) \tag{6}$$

for every arbitrary sequence $\{\alpha_i\}$ or $\{\beta_j\}$ satisfying Eq. (2).

Obviously, both $\{\alpha_i^*\}$ and $\{\beta_j^*\}$ are functions of Δ_e and Δ_p, thus $V_m = V_m(\Delta_e,\Delta_p)$.

The mixed guidance law synthesis problem, which is the subject of the present paper, is formulated as follows. For a given but otherwise arbitrary pure strategy set of the evader Δ_e, find the optimal pure strategy set of the pursuer $\tilde{\Delta}_p$, which satisfies

$$V_m(\Delta_e,\tilde{\Delta}_p) \geq V_m(\Delta_e,\Delta_p) \tag{7}$$

for every admissible Δ_p.

Method of Solution

Basically, the problem is solved by forming a sequence of pure strategy sets $\{\tilde{\Delta}_p^{(k)}\}_{k=1,2,...}$ which converges to $\tilde{\Delta}_p$. The set $\tilde{\Delta}_p^{(k)}$, which is composed of k elements, satisfies the inequality

$$V_m(\Delta_e,\tilde{\Delta}_p^{(k)}) \geq V_m(\Delta_e,\Delta_p) \tag{8}$$

for any admissible Δ_p with k or less elements and is called a "k optimal" set.

The important features of the solution algorithm, developed in Ref. 8, are summarized here for the sake of completeness.

First it is necessary to introduce several definitions. A game in which the pure strategy sets of the pursuer and the evader are Δ_p and Δ_e, respectively, will be referred to as a "pair (Δ_e,Δ_p)." The entries of the payoff matrices and the corresponding optimal mixed strategies for the pair (Δ_e,Δ_p) and $(\Delta_e,\tilde{\Delta}_p^{(k)})$ will de denoted by P_{ij}, $\{\alpha_i^*\}$, $\{\beta_j^*\}$ and $\tilde{P}_{ij}^{(k)}$, $\{\tilde{\alpha}_i^{(k)}\}$, $\{\tilde{\beta}_j^{(k)}\}$, respectively. A pure strategy set that is composed of k elements will be denoted by $\Delta_p^{(k)}$.

The properties that must be satisfied by $\tilde{\Delta}_p^{(k)}$ are as follows:

$$\sum_{i=1}^{m_e} \tilde{\alpha}_i^{(k)}\tilde{P}_{ij}^{(k)} = V_m(\Delta_e,\tilde{\Delta}_p^{(k)}) \ \forall \ 1 \leq j \leq k \tag{P1}$$

$$\tilde{\Delta}_p^{(k)} = \arg\max_{\Delta_p^{(k)}} \{ \min_{1\leq i\leq m_e} \sum_{j=1}^{k} \beta_j^*(\Delta_p^{(k)})P_{ij}(\Delta_p^{(k)})\} \tag{P2}$$

Property P1 states that $\tilde{\Delta}_p^{(k)}$ should be composed of k guidance policies that guarantee the *same* kill probability (cost) against the evader's optimal mixed strategy.

Property P2 states that $\tilde{\Delta}_p^{(k)}$ should be such that the kill probability guaranteed by it be the maximum obtainable with strategy sets of k elements.

The iterative algorithm, used in the solution of the examples, is given by:

Step 1—set $k=1$

Step 2—find $\tilde{\Delta}_p^{(k)}$ that satisfied P1 and P2.

Step 3—solve the pair $(\Delta_e,\tilde{\Delta}_p^{(k)})$ yielding $\{\tilde{\alpha}_i^{(k)}\}$, $\{\tilde{\beta}_j^{(k)}\}$ and $V_m(\Delta_e,\tilde{\Delta}_p^{(k)})$, [$\{\tilde{\alpha}_i^{(k)}\}$, $\{\tilde{\beta}_j^{(k)}\}$ may not be unique].

Step 4—for every $\{\tilde{\alpha}_i^{(k)}\}$ search for a strategy $\delta_{p_l} \notin \tilde{\Delta}_p^{(k)}$ such that $\sum_{i=1}^{m_e} \tilde{\alpha}_i^{(k)}P_{il} > V_m(\Delta_e,\tilde{\Delta}_p^{(k)})$. If there is any $\{\tilde{\alpha}_i^{(k)}\}$ against which no such δ_{p_l} is found then $\tilde{\Delta}_p = \tilde{\Delta}_p^{(k)}$ and go to step 6, otherwise go to step 5.

Step 5—if $k=1$ set $k=k+1$ then go to step 2, otherwise check if $V_m(\Delta_e,\tilde{\Delta}_p^{(k)}) - V_m(\Delta_e,\tilde{\Delta}_p^{(k-1)}) > \delta$ for a given δ. If so, set $k=k+1$ and go to step 2, otherwise go to step 6.

Step 6—stop.

Steps 2 and 4 require a considerable computational effort since they have to be implemented by various search methods. In Ref. 8 it is also shown that $\tilde{\Delta}_p^{(k)}$ converges to $\tilde{\Delta}_p$.

On the Evader's Pure Strategy Set

As previously mentioned, the pure strategy set of the evader Δ_e is an infinite set of "actions" each of which is composed of a maneuver sequence and eventually of an ECM policy. An intelligent target must perform a "high energy" maneuver in an unpredictable way. By "high energy" maneuvers we mean the maneuvers with which it is most difficult for the missile to cope. In the ECM-free situation, it would roughly mean a maneuver that is performed at the evader's maximum lateral acceleration and that causes significant deviations from a straight-line course.

Unpredictability and the requirement for "high energy" maneuvers are somewhat conflicting. It can be shown that the "maximum energy" requirement leads to periodic maneuvers. This requirement conflicts with unpredictability because periodic maneuvers become predictable after an observation period. On the other hand, the unpredictability requirement leads to random maneuvers that, in terms of energy, are not as effective, because they spread some of their energy at high frequencies. This means that the evader's optimal strategy is

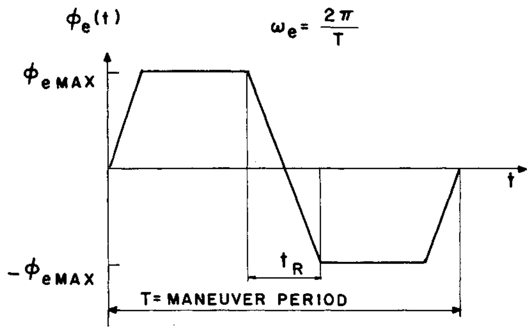

Fig. 2 Shape of the evader's maneuver.

some sort of compromise between these conflicting requirements.

Let us assume that the missile designer has accepted the rules of the game about the "choice" of a strategy at the outset as the only plausible way to play in order to guarantee some average performance. Paradoxically, in such a case, the most difficult task for the missile would be to hit a periodically maneuvering target. This means that the missile designer should design his missile against a subset of Δ_e, say Δ'_e, which is composed of "high energy" periodic maneuvers.

In the examples that follow it is assumed that the target maneuvers in the horizontal plane. Its lateral acceleration a_e is given by:

$$a_e = \begin{cases} g \tan \phi_e & \omega_e \neq 0 \\ \pm g \tan \phi_{e_{\max}} & \omega_e = 0 \end{cases} \tag{9}$$

where g is the acceleration of gravity and ϕ_e is the bank angle which varies periodically at frequency ω_e (as shown in Fig. 2) and which has random phase. $\phi_{e_{\max}}$ is the maximum bank angle, and it is given by

$$\phi_{e_{\max}} = \cos^{-1}\left(\frac{g}{a_{e_{\max}}}\right) \tag{10}$$

where $a_{e_{\max}}$ is the aircraft's lateral acceleration limit. In Fig. 2 the constrained roll dynamics of the aircraft is taken into account in the average sense. The time required to change the bank angle by $2\phi_{e_{\max}}$ is t_R.

Since the electronic jinking causes a movement of the aircraft's center of reflection from wing tip to wing tip, it is clear that the disturbance caused by it in the horizontal plane is a function of the bank angle ϕ_e. In this paper we approximate the horizontal disturbance perpendicular to the reference line by:

$$w = w_{\max} \operatorname{sign}[\sin(\omega_J t + \phi_J)] \cos\phi_e \tag{11}$$

where $w_{\max}$ is the semispan of the aircraft and ω_J the frequency of the electronic jinking. The random phase is ϕ_J. Note that the electronic jinking is considered as a periodic random phase disturbance with modulated amplitude. The amplitude decreases with increasing bank angle.

In the first example, which deals with an ECM-free environment, Δ'_e is composed of maneuvers at 17 different frequencies equally distributed on [0.0, 4.0] rad/s. In the second example, Δ'_e is composed of a straight-flying trajectory and of maneuvers at five different frequencies equally distributed on [0.0, 4.0] rad/s combined with seven jinking frequency options equally distributed on [1.0, 4.0] rad/s. Therefore, Δ'_e has a total of 42 elements. In this example, the respective random phases of the kinematic maneuver and of the electronic jinking are taken to be uncorrelated.

Note that even though the electronic jinking can be applied at high frequencies, we limit our attention to frequencies up to 4 rad/s. This is done because any missile designer, who is aware of the existence of disturbances at high frequencies, will undoubtedly make use of a prefilter that "prevents" signals at high frequencies from entering the system. The value of 4 rad/s was chosen as a limit because the deviations from the straight line caused by a maneuver at this frequency are small enough (30 cm in this example) and can be "filtered out" of the system along with all the disturbances at high frequencies without any harm.

On the Missile's Pure Strategy Set

In order to avoid confusion, we adopt in this paper the following terminology. A "guidance law" will be understood to be a function that maps the estimated state into acceleration commands, regardless of the form of the estimator used in the guidance loop. The combination between a guidance law and an estimator will be referred to as a "guidance policy."

The designer's task is to find the optimal pure strategy set of the missile, i.e., the number and the type of guidance policies that should be programmed into the missile in order to obtain optimum performance. This task is quite a formidable one in view of the endless possible structures and combinations of guidance laws and estimators.

A practical and reasonable approach to the solution of this problem is to adopt specific structures for the guidance law and the estimator, to determine the dimension of the estimator, and to search within these limitations.

In the examples that follow, the structure adopted for the guidance law is given by[1]:

$$\delta_{p_j} = \begin{cases} \dfrac{N'(T,\mu)}{t_{go}^2 \cos\gamma_p} ZEM_j & T > T_s \\ a_{p_{\max}} \cdot \operatorname{sign}(ZEM_j) & T \leq T_s \end{cases} \tag{12}$$

where ZEM_j is the zero effort miss term given by

$$ZEM_j = \hat{Y}^{(j)} + t_{go}\hat{\dot{Y}}^{(j)} - \tau_p^2(e^{-T} + T - 1)a_p \cos\gamma_p \tag{13}$$

$$T = t_{go}/\tau_p \tag{14}$$

where T_s is the value of T at which the control switches from linear to bang-bang and N' is the effective navigation gain given by

$$N'(T,\mu) = \frac{2}{(1 - 1/\mu) - 2(e^{-T} + T - 1)/T^2} \tag{15}$$

$$\mu = a_{p_{\max}}/a_{e_{\max}} \tag{16}$$

T_s is the solution of

$$(1 - 1/\mu) - 2(e^{-T_s} + T_s - 1)/T_s^2 = 0 \tag{17}$$

The missile's lateral acceleration limit is $a_{p_{\max}}$, a_p is its actual lateral acceleration, γ_p is its flight-path angle, and τ_p is the equivalent time constant of the autopilot dynamics. In this paper we assume that the missile's autopilot response can be represented by a first-order time constant.

The estimates of y and $\dot{y}$, obtained for the jth estimator, are $\hat{y}^{(j)}$ and $\hat{\dot{y}}^{(j)}$, respectively. The maximum dimension of the estimators is selected to be four or six depending on whether or not the electronic jinking is taken into account. Their structure, given in Fig. 3, is of a steady-state Kalman filter based on the assumption that the target's maneuver and jinking processes can be approximated by the following shaping filters: (in the sense presented in Refs. 10 and 11).

$$G_e = \frac{k_e(s + \alpha_e)}{s^2 + 2\zeta_e\tilde{\omega}_e s + \tilde{\omega}_e^2} \tag{18}$$

fed by a white noise of density $\phi_e = a_{e_{\max}}^2$ for the maneuver process and

$$G_J = \frac{k_J(s + \alpha_J)}{s^2 + 2\zeta_J\tilde{\omega}_J s + \tilde{\omega}_J^2} \tag{19}$$

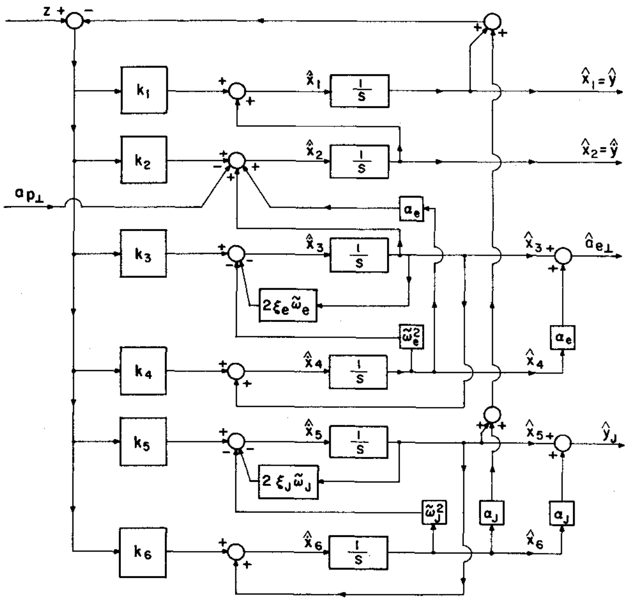

Fig. 3 Estimator structure.

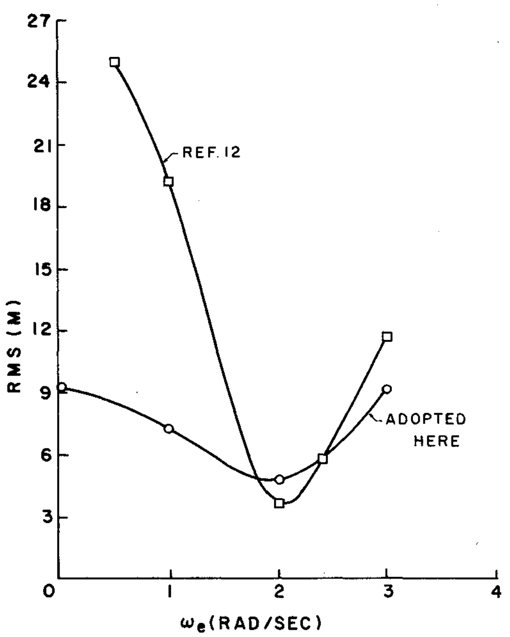

Fig. 4 Sensitivity comparison between guidance policies.

fed by a white noise of density $\phi_J = W_{\max}^2$ for the electronic jinking process.

The parameters at the designer's disposal are k_e, α_e, $\tilde{\omega}_e$, ζ_e and k_J, α_J, $\tilde{\omega}_J$, ζ_J. Different parameters imply different assumptions on the target's motion and consequently lead to different estimators (of the same structure), which in turn lead to different guidance policies in Δ_p.

Remark 1: In this paper we have adopted the common practice in the missile community of making use of guidance laws that are derived for a perfect information environment by replacing the state variables by their estimates. Although in general this is not optimal, it is very attractive for implementation.

Remark 2: The guidance law given by Eqs. (12–17) is optimal in the sense that it is the saddle point solution of the perfect information differential game.[1] It was preferred in comparison with other "modern" guidance laws[12] based on one-sided optimization, because it is less sensitive to miss modeling of the evader's motion as can be seen from Fig. 4. The guidance policies of Fig. 4 are based on the assumption

Table 1 Parameter values for the examples

Initial geometry	Missile's dynamics
$R_o = 4500$ m	$\tau_p = 0.2$ s
$\gamma_{po} = 0$ rad	Lethality model
$\gamma_{eo} = \pi$ rad	$R_e = 3$ m
Kinematic constants	Glint
$V_p = 600$ m/s	B.W. = 2 Hz
$V_e = 300$ m/s	σ (Ex. 1) = 2.5 m
$a_{p_{\max}} = 150$ m/s^2	σ (Ex. 2) = 0
$a_{e_{\max}} = 50$ m/s^2	
A/C's dynamics	Electronic jinking (Ex. 2)
$t_R = 2$ s	$w_{\max} = 4.7$ m

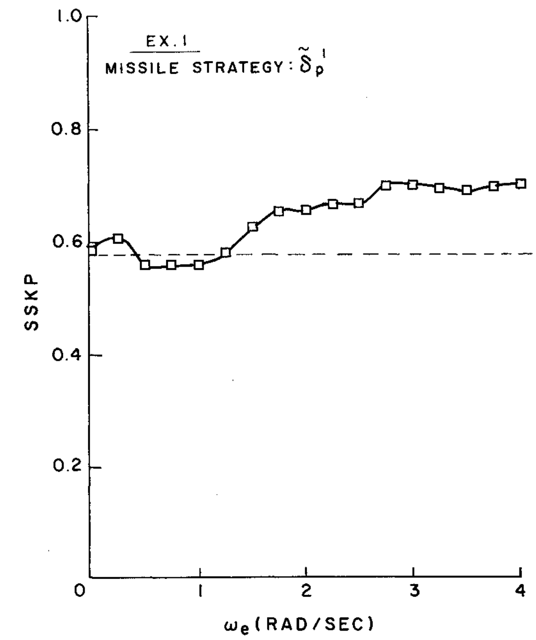

Fig. 5 Performance of near-optimal guidance policy in ECM-free environment.

that the target maneuvers sinusoidally at a frequency $\omega_e = 2$ rad/s. The estimator structure used in this case is given in Ref. 7.

Remark 3: The estimator structure given in Fig. 3 was found to be optimal (within the dimension limitations) after intensive work with simpler shaping filters of dimensions 1 and 2.

Examples

The examples are based on the specific parameter values presented in Table 1.

The warhead lethality model in both examples is

$$p[x_1(t_f)] = \begin{cases} 0.9 & |x_1(t_f)| \le R_e \\ 0.9 \exp\left\{-4\left(\dfrac{|x_1(t_f)|}{R_e} - 1\right)^2\right\} & |x_1(t_f)| > R_e \end{cases}$$

where x_1 is the miss distance and R_e a parameter that is indicative of the missile's lethal radius.

All the results in this section are based on averages of 200 Monte Carlo runs for each point.

Example 1: ECM-free Environment

In this example we search for the optimal missile guidance strategy against the previously defined pure strategy set of the evader Δ_e' without the ECM option.

The first step in the solution procedure is to find $\tilde{\Delta}_p^{(1)} = [\delta_p^1]$, i.e., the optimal *pure* strategy (of the predetermined structure). Search in the parameter space $(k_e, \alpha_e, \tilde{\omega}_e, \zeta_e)$ led to the

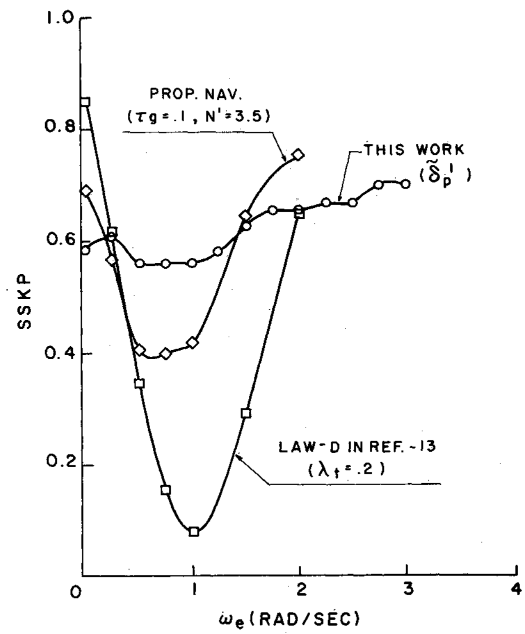

Fig. 6 "Weak spots" in performance of guidance laws previously discussed in the literature.

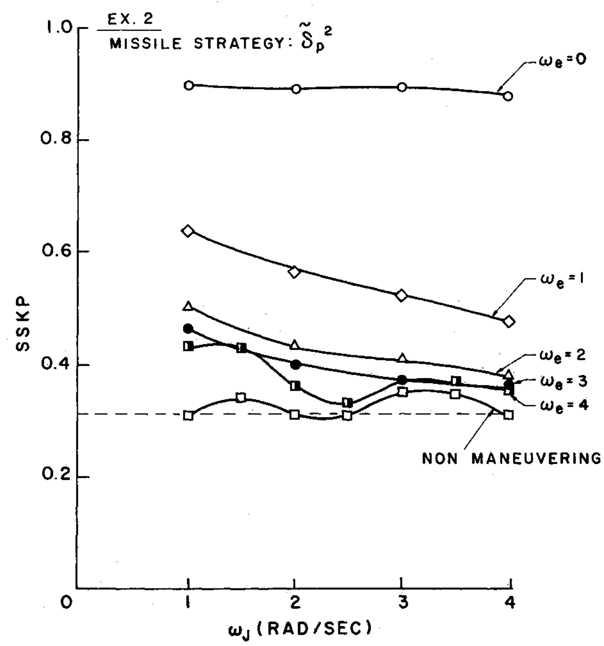

Fig. 8 Performance of optimal pure guidance policy in electronic jinking environment.

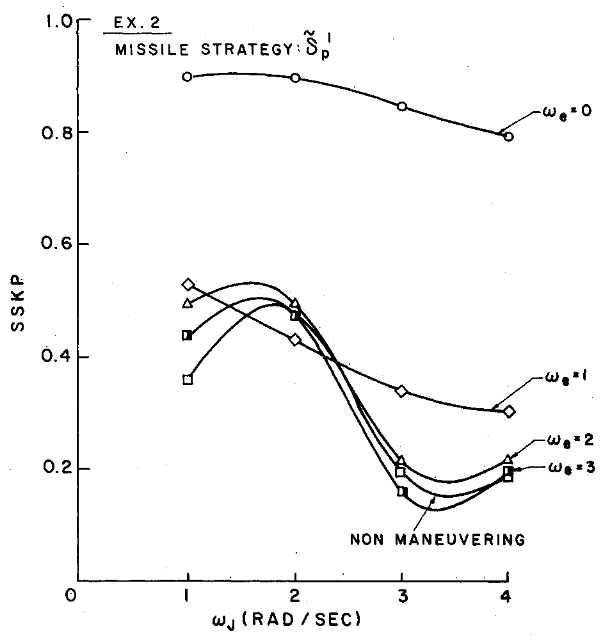

Fig. 7 Effects of electronic jinking on performance of guidance policy designed for ECM-free environment.

optimal pure strategy δ_p^1 given by the following: $k_e = 0.55$, $\alpha_e = 2.0$, $\tilde{\omega}_e = 0.5$, $\zeta_e = 0.2$.

The performance of this guidance policy is shown in Fig. 5. From this figure it can be seen that, in the maneuver frequency range $0 \le \omega_e < 1.25$ rad/s, this guidance policy approximately has a uniform performance against all high-energy maneuvers. The performance against maneuvers at higher frequencies $\omega_e > 1.5$ rad/s gradually improves as frequency increases. At these "high" frequencies the lateral displacements of the aircraft relative to the reference line decrease progressively, thus, the evasive energy of the maneuver also decreases.

The second step in the solution procedure is to compute the evader's optimal strategy against $\tilde{\Delta}_p^{(1)}$, say $\{\tilde{\alpha}_i^{(1)}\}$, and to search for another pure guidance strategy that performs better than $\tilde{\Delta}_p^{(1)}$ against $\{\tilde{\alpha}_i^{(1)}\}$. It was shown[8] that if such a strategy exists then it is possible to improve the guaranteed average performance of the missile by a set of two guidance policies $\tilde{\Delta}_p^{(2)}$. If such a strategy cannot be found, then $\tilde{\Delta}_p^{(1)}$ is the optimal strategy set, i.e., $\tilde{\Delta}_p = \tilde{\Delta}_p^{(1)}$.

In the present example, the target's optimal mixed strategy, $\{\tilde{\alpha}_i^{(1)}\}$, is a uniform distribution over the range ω_e: [0.5, 1.25] rad/s, which guarantees a single-shot kill probability (SSKP) of approximately 0.56 (see Fig. 5). Search in the parameter space did not yield any guidance policy that performs better than $\tilde{\Delta}_p^{(1)}$ against the optimal strategy $\{\tilde{\alpha}_i^{(1)}\}$. This result leads to the conclusion that, in this particular case, the optimal missile strategy is *pure*, i.e., $\tilde{\Delta}_p = \tilde{\Delta}_p^{(1)}$.

It is important to point out that this "pure" optimal guidance policy found by the *mixed strategy approach* has no "weak spots" as compared to PROP. NAV. or to other modern guidance policies discussed in the literature,[13] as can be seen from Fig. 6. This no-weak-spot situation is a result of property P2 mentioned earlier. Property P2 actually drives to guidance strategies with performance as flat as possible (see Fig. 5) given the constraints imposed by the selected structure of the guidance policies considered. Since this property is imbedded in the solution, the design procedure followed here always leads to guidance strategies that approximate as closely as possible such a no-weak-spot situation.

Example 2: ECM Environment

In this example we search for the optimal missile guidance strategy against the previously defined Δ_e' with the electronic jinking option.

Figure 7 presents the performance of the optimal guidance policy found in the previous example against an electronically jinking target. From this figure it is clear that the performance of an optimal guidance policy designed to operate in an ECM-free environment is very much degraded by electronic jinking. It is, therefore, necessary to take into account the eventual existence of electronic jinking in the design procedure of guidance policies.

As in example 1, the first step in the design procedure is to find $\tilde{\Delta}_p^{(1)} = [\delta_p^2]$. After a search in the parameter space $(k_e, \alpha_e, \tilde{\omega}_e, \zeta_e, k_J, \alpha_J, \tilde{\omega}_J, \zeta_J)$ the optimal pure strategy δ_p^2 was found to be given by: $k_e = 1.0$, $\alpha_e = 1.5$, $\tilde{\omega}_e = 0.5$, $\zeta_e = 0.2$, $k_J = 1.0$, $\alpha_J = 8.0$, $\tilde{\omega}_J = 2.5$, $\zeta = 0.6$. The performance of this guidance policy is shown in Fig. 8. From the figure it can be seen that the guaranteed kill probability is $V_m(\Delta_e, \tilde{\Delta}_p^{(1)}) = 0.31$. The optimal strategy of the evader against $\tilde{\Delta}_p^{(1)}$, $\{\tilde{\alpha}_i^{(1)}\}$, is to fly in a

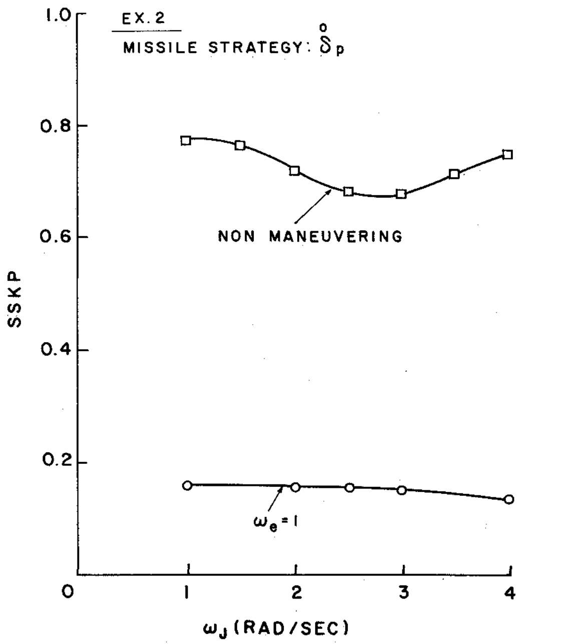

Fig. 9 Performance of $\overset{\circ}{\delta}_p$ against $\{\tilde{\alpha}_i^{(1)}\}$ in electronic jinking environment.

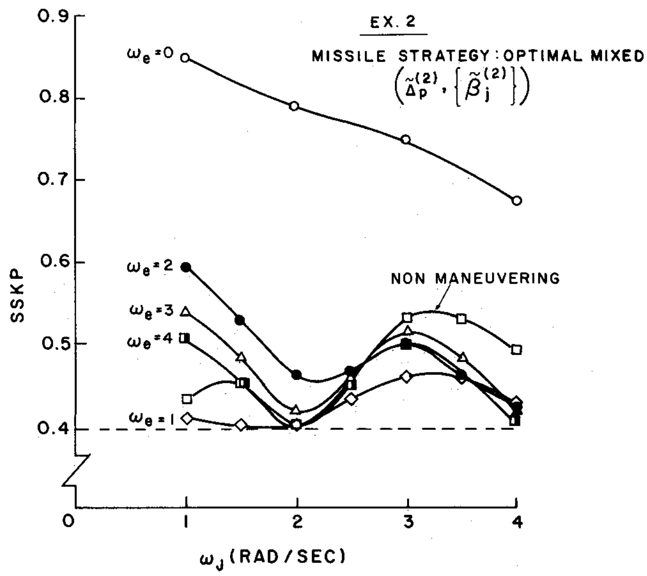

Fig. 10 Performance of optimal mixed guidance policy in electronic jinking environment.

straight line and to apply electronic jinking at frequencies 1.0, 2.0, 2.5 or 4.0 rad/s.

Note from Figs. 7 and 8 that a constant evasive maneuver ($\omega_e = 0.0$) in this two-dimensional situation is not effective. This is because the amplitude of the disturbance caused by the electronic jinking in the horizontal plane is very small.

The second step in the design procedure is to search for another pure guidance strategy $\overset{\circ}{\delta}_p$, which performs better than $\tilde{\Delta}_p^{(1)}$ against $\{\tilde{\alpha}_i^{(1)}\}$. Such a strategy exists, as can be seen from Fig. 9. Its parameters are $k_e = 1.0$, $\alpha_e = 1.5$, $\bar{\omega}_e = 0.5$, $\zeta_e = 0.2$, $k_J = 1.0$, $\alpha_J = 35.0$, $\bar{\omega}_J = 0.5$, $\zeta_J = 0.2$. This means that there exists an optimal pure strategy set composed of two guidance policies $\tilde{\Delta}_p^{(2)}$, which performs better than $\tilde{\Delta}_p^{(1)}$.

As a result of a search in the parameter space, $\tilde{\Delta}_p^{(2)}$ was found. Its two guidance policies, δ_{p_1} and δ_{p_2}, are given by: $k_e = 1.0$, $\alpha_e = 1.5$, $\bar{\omega}_e = 0.5$, $\zeta_e = 0.2$, $k_J = 1.0$, $\alpha_J = 16.5$, $\bar{\omega}_J = 0.9$, $\zeta_J = 0.2$ and $k_e = 1.0$, $\alpha_e = 1.5$, $\bar{\omega}_e = 0.5$, $\zeta_e = 0.2$, $k_J = 1.0$, $\alpha_j = 12.0$, $\bar{\omega}_J = 2.8$, $\zeta_J = 0.3$, respectively. The optimal mixed strategy of the missile in this case is $\{\tilde{\beta}_j^{(2)}\} = \text{col}(0.5,0.5)$. The performance of this optimal mixed guidance strategy is shown in Fig. 10. From Fig. 10 it can be seen that, in this case, the guaranteed SSKP is $V_m(\Delta_e, \tilde{\Delta}_p^{(2)}) = 0.4$. This is an improvement of approximately 30% as compared to the performance guaranteed by $\tilde{\Delta}_p^{(1)}$.

The optimal strategy of the evader $\{\tilde{\alpha}_i^{(2)}\}$, in this case, is a distribution on the following five elements: 1) straight flight with jinking at $\omega_J = 2$ rad/s; 2) and 3) maneuvering at $\omega_e = 1$ rad/s while jinking at $\omega_J = 1.5$ or 2.0 rad/s; and 4) and 5) maneuvering at $\omega_e = 4$ rad/s while jinking at $\omega_J = 2$ or 4 rad/s.

The next step is to search for a pure guidance strategy that performs better than $\tilde{\Delta}_p^{(2)}$ against $\{\tilde{\alpha}_i^{(2)}\}$. The search for such a guidance policy in the parameter space did not yield further improvement. It can be concluded with reasonable confidence that such a guidance policy may not exist within the limitations imposed by the structures adopted in this paper. Thus, $\Delta_p = \tilde{\Delta}_p^{(2)}$.

Note that the performance of mixed guidance strategies is always superior or equal to the performance of pure strategies since by definition[3] a pure strategy is a particular case of a mixed strategy.

Conclusions

This paper summarizes a major phase in the research effort oriented to the synthesis of improved guidance laws by a mixed strategy approach. Two interesting applications of this method are presented. In the first example, which deals with an ECM-free environment, the optimal guidance strategy turns out to be pure. However, it is demonstrated that this pure guidance strategy, obtained by the mixed strategy approach, performs better than any other guidance law discussed in the literature. In the second example, where the evader used electronic jinking during its maneuver, the optimal guidance strategy is truly mixed. The optimal pure strategy set for the missile in this case is composed of only two guidance policies. Both examples indicate that the design procedure based on the mixed strategy concept yields improved missile performance. The origin of the improvements obtained by the approach of mixed guidance strategies is in the "no-weak-spot" property discussed in connection with Fig. 6. This property is not influenced by whether or not the optimal strategy is truly mixed or turns out to be pure.

Two parameters have a major influence on the outcome of the design process: the missile/target maneuver ratio and the lethal range of the missile warhead. It is clear that a very powerful missile, having a high acceleration ratio and a large warhead, has no need for a complex guidance strategy. At the other end, even the best guidance policy cannot transform a sluggish missile with a small warhead into an efficient weapon. Between these two extremes there is a large domain of missile design parameters, where the inherent improvements resulting from the mixed strategy approach can be translated into terms of increased kill probability.

This paper concentrated on the two-dimensional linearized version of the missile-aircraft end game. Although the validity of the mixed strategy approach is not limited to this case, the results for other cases have to be verified by more complex three-dimensional nonlinear simulations.

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